LOGARITMOS – 4 ESO – EJERCICIOS RESUELTOS

PROBLEMA 1.- Calcular los siguientes logaritmos usando la definición.

a) $log_2 64$ b) $log_2 16$ c) $log_2 \frac{1}{4}$ d) $log_2 \sqrt{2}$

e) $log_3 243$ f) $log_3 \frac{1}{27}$ g) $log_3 \sqrt[3]{9}$ h) log 0,001

i) $log_3 729$ j) $log_3 \frac{1}{9}$ k) $log_3 \sqrt[3]{81}$ l) $log_2 32$

a) $\log_2 64 = x \rightarrow 64 = 2^x \rightarrow 2^6 = 2^x \Rightarrow \boxed{x = 6}$

b) $\log_2 16 = x \rightarrow 16 = 2^x \rightarrow 2^4 = 2^x \rightarrow \boxed{x = 4}$

c) $\log_2 \frac{1}{4} = x \rightarrow \frac{1}{4} = 2^x \rightarrow 4^{-1} = 2^x \rightarrow (2^2)^{-1} = 2^x \rightarrow 2^{-2} = 2^x \rightarrow \boxed{x = -2}$

d) $\log_2 \sqrt{2} = x \rightarrow \sqrt{2} = 2^x \rightarrow 2^{1/2} = 2^x \Rightarrow \boxed{x = 1/2}$

e) $\log_3 243 = x \rightarrow 243 = 3^x \rightarrow 3^5 = 3^x \Rightarrow \boxed{x = 5}$

$243 = 3^5$

243	3
81	3
27	3
9	3
3	3
1	

f) $\log_3 \frac{1}{27} = x \rightarrow \frac{1}{27} = 3^x$

$\frac{1}{3^3} = 3^x \rightarrow 3^{-3} = 3^x \Rightarrow \boxed{x = -3}$

g) $\log_3 \sqrt[3]{9} = x \rightarrow \sqrt[3]{9} = 3^x \rightarrow \sqrt[3]{3^2} = 3^x$

$3^{2/3} = 3^x \Rightarrow \boxed{x = 2/3}$

h) $\log 0{,}001 = x \rightarrow 0{,}001 = 10^x \rightarrow 10^{-3} = 10^x \rightarrow \boxed{x = -3}$

i) $\log_3 729 = x \rightarrow 729 = 3^x \rightarrow 3^6 = 3^x \rightarrow \boxed{x = 6}$

729	3
243	3
81	3
27	3
9	3
3	3
1	

$\Rightarrow 729 = 3^6$

j) $\log_3 \frac{1}{9} = x \rightarrow \frac{1}{9} = 3^x \rightarrow \frac{1}{3^2} = 3^x$

$3^{-2} = 3^x \rightarrow \boxed{x = -2}$

k) $\log_3 \sqrt[3]{81} = x \rightarrow \sqrt[3]{81} = 3^x \rightarrow \sqrt[3]{3^4} = 3^x$

$3^{4/3} = 3^x \Rightarrow \boxed{x = 4/3}$

l) $\log_2 32 = x \rightarrow 32 = 2^x \rightarrow 2^5 = 2^x \Rightarrow \boxed{x = 5}$

PROBLEMA 2.- Calcula la base de los siguientes logaritmos.

a) $log_b 10000 = 2$ b) $log_b 125 = 3$ c) $log_b \frac{1}{4} = -1$ d) $log_x \sqrt{2} = \frac{1}{2}$

e) $log_x 16 = 2$ f) $log_x 9 = 4$ g) $log_x \sqrt[3]{4} = \frac{2}{3}$ h) $log_x 32 = 5$

a) $\log_b 10000 = 2 \rightarrow 10000 = b^2 \rightarrow 100^2 = b^2 \rightarrow \boxed{b = 100}$

b) $\log_b 125 = 3 \rightarrow 125 = b^3 \rightarrow 5^3 = b^3 \rightarrow \boxed{b = 5}$

c) $\log_b \frac{1}{4} = -1 \rightarrow \frac{1}{4} = b^{-1} \rightarrow 4^{-1} = b^{-1} \rightarrow \boxed{b = 4}$

d) $\log_b \sqrt{2} = \frac{1}{2} \rightarrow \sqrt{2} = b^{1/2} \rightarrow 2^{1/2} = b^{1/2} \rightarrow \boxed{b = 2}$

e) $\log_x 16 = 2 \rightarrow 16 = x^2 \rightarrow 4^2 = x^2 \rightarrow \boxed{x = 4}$

f) $\log_x 9 = 4 \rightarrow 9 = x^4 \rightarrow (\sqrt{3})^4 = x^4 \rightarrow \boxed{x = \sqrt{3}}$

g) $\log_x \sqrt[3]{4} = \frac{2}{3} \rightarrow \sqrt[3]{4} = x^{2/3} \rightarrow \sqrt[3]{2^2} = x^{2/3} \rightarrow 2^{2/3} = x^{2/3} \rightarrow \boxed{x = 2}$

h) $\log_x 32 = 5 \rightarrow 32 = x^5 \rightarrow 2^5 = x^5 \rightarrow \boxed{x = 2}$

PROBLEMA 3.- Calcula aplicando la definición de logaritmo:

$$log_4 16^3 + log_4 2 + log 0{,}0001 + log \frac{\sqrt[3]{10}}{100}$$

$\log_4 16^3 = x \rightarrow 16^3 = 4^x \Rightarrow (4^2)^3 = 4^x \rightarrow 4^6 = 4^x \rightarrow \boxed{x = 6}$

$\log_4 2 = x \rightarrow 2 = 4^x \rightarrow 2 = (2^2)^x \rightarrow 2 = 2^{2x} \rightarrow 2x = 1 \rightarrow \boxed{x = \frac{1}{2}}$

$\log 0{,}0001 = x \rightarrow 0{,}0001 = 10^x \rightarrow \frac{1}{10^4} = 10^x \rightarrow 10^{-4} = 10^x \rightarrow \boxed{x = -4}$

$\log \frac{\sqrt[3]{10}}{100} = x \rightarrow \frac{\sqrt[3]{10}}{100} = \rightarrow 10^{1/3} \cdot 10^{-2} = 10^x \rightarrow 10^{-5/3} = 10^x \rightarrow \boxed{x = -\frac{5}{3}}$

$6 + \frac{1}{2} - 4 - \frac{5}{3} = \boxed{\frac{5}{6}}$

PROBLEMA 4.- Utiliza las propiedades de los logaritmos para obtener el valor de las siguientes expresiones:

a) $log\,20 + log\,50$ b) $log_2 400 - log_2 25$ c) $log_2 288 - 2 \cdot log_2 6$

d) $log\,4 + log\,0{,}025$ e) $log_3 0{,}2 + log_3 405$ f) $log_5 25 + log_5 125$

a) $\log 20 + \log 50 = \log(20 \cdot 50) = \log 1000 = \log 10^3 = 3$

b) $\log_2 400 - \log_2 25 = \log_2 \frac{400}{25} = \log_2 16 = \log_2 2^4 = 4$

c) $\log_2 288 - 2 \cdot \log_2 6 = \log_2 288 - \log_2 6^2 = \log_2 \frac{288}{36} =$

$= \log_2 8 = \log_2 2^3 = 3$

d) $\log 4 + \log 0{,}025 = \log(4 \cdot 0{,}025) = \log 0{,}1 = \log 10^{-1} = -1$

e) $\log_3 0{,}2 + \log_3 405 = \log_3(0{,}2 \cdot 405) = \log_3 81 = \log_3 3^4 = 4$

f) $\log_5 25 + \log_5 125 = \log_5(25 \cdot 125) = \log_5 5^5 = 5$

PROBLEMA 5.- Si log x = 1,3 y log y = 0,8, calcula:

a) *log* (x·y) b) $log\ (x \cdot \sqrt{y})$ c) $log\ \frac{y}{x^2}$ d) $log\ \sqrt{\frac{x}{y}}$

a) $\log(x \cdot y) = \log x + \log y = 1{,}3 + 0{,}8 = 2{,}1$

b) $\log(x \cdot \sqrt{y}) = \log x + \log \sqrt{y} = \log x + \log y^{1/2} = \log x + \frac{1}{2}\log y =$

$= 1{,}3 + \frac{1}{2} \cdot 0{,}8 = 1{,}3 + 0{,}4 = 1{,}7$

c) $\log \frac{y}{x^2} = \log y - \log x^2 = \log y - 2 \cdot \log x = 0{,}8 - 2 \cdot 1{,}3 =$

$= 0{,}8 - 2{,}6 = -1{,}8$

d) $\log \sqrt{\frac{x}{y}} = \log\left(\frac{x}{y}\right)^{1/2} = \frac{1}{2}\log\frac{x}{y} = \frac{1}{2}(\log x - \log y) =$

$= \frac{1}{2}(1{,}3 - 0{,}8) = \frac{1}{2} \cdot 0{,}5 = 0{,}25$

PROBLEMA 6.- Si $A = \frac{8x^2}{\sqrt{y}}$, calcular $log_2 A$ sabiendo que $log_2 x = 1{,}5$ y $log_2\, y = -0{,}6$

$$\log_2 \frac{8x^2}{\sqrt{y}} = \log_2 8\cdot x^2 - \log_2 \sqrt{y} = \log_2 8 + \log_2 x^2 - \log_2 \sqrt{y} =$$
$$= \log_2 2^3 + 2\cdot \log_2 x - \log_2 y^{1/2} = 3 + 2\cdot 1{,}5 - \tfrac{1}{2}\log y =$$
$$= 3+3+2\cdot 0{,}6 = \boxed{6{,}6}$$

PROBLEMA 7.- Sabiendo que el log A = -1,2, log B = 0,7 y log C = 2,3, calcular utilizando las propiedades de los logaritmos:

a) $\log \frac{A\cdot B}{10C}$ b) $\log \frac{A^2}{BC}$ c) $\log \sqrt{\frac{1000\cdot A}{B}}$ d) $\log \frac{\sqrt{A}\cdot B^2}{C^3}$

a) $\log \frac{A\cdot B}{10C} = \log A\cdot B - \log 10\cdot C = \log A + \log B - (\log 10 + \log C) =$
$= \log A + \log B - 1 - \log C = -1{,}2 + 0{,}7 - 1 - 2{,}3 = \boxed{3{,}8}$

b) $\log \frac{A^2}{B\cdot C} = \log A^2 - \log B\cdot C = 2\cdot \log A - (\log B + \log C) =$
$= 2\cdot(-1{,}2) - (0{,}7 + 2{,}3) = -2{,}4 - 3 = \boxed{-5{,}4}$

c) $\log \sqrt{\frac{1000\cdot A}{B}} = \log \left(\frac{1000A}{B}\right)^{1/2} = \frac{1}{2}\log \frac{1000A}{B} =$
$= \frac{1}{2}(\log 1000A - \log B) = \frac{1}{2}(\log 1000 + \log A - \log B) =$
$= \frac{1}{2}(3 + (-1{,}2) - 0{,}7) = \frac{1}{2}\cdot 1{,}1 = \boxed{0{,}55}$

d) $\log \frac{\sqrt{A}\cdot B^2}{C^3} = \log \sqrt{A}\cdot B^2 - \log C^3 = \log \sqrt{A} + \log B^2 - \log C^3 =$
$= \log A^{1/2} + \log B^2 - \log C^3 = \frac{1}{2}\log A + 2\log B - 3\log C =$
$= \frac{1}{2}(-1{,}2) + 2\cdot 0{,}7 - 3\cdot(2{,}3) = -0{,}6 + 1{,}4 - 6{,}9 = \boxed{-6{,}1}$

PROBLEMA 8.- Hallar el valor de x en cada caso aplicando las propiedades de los logaritmos.

a) log x = log 25 – log 3 b) log x = 2 log 3 – 1 c) Ln x = 3 Ln 6 +1/2 Ln5

a) $\log x = \log 25 - \log 3 \rightarrow \log x = \log \frac{25}{3} \Rightarrow \boxed{x = \frac{25}{3}}$

b) $\log x = 2 \cdot \log 3 - 1 \rightarrow \log x = \log 3^2 - \log 10$

$\log x = \log \frac{9}{10} \Rightarrow \boxed{x = \frac{9}{10}}$

c) $\text{Ln}\, x = 3 \cdot \text{Ln}\, 6 + \frac{1}{2} \text{Ln}\, 5$

$\text{Ln}\, x = \text{Ln}\, 6^3 + \text{Ln}\, \sqrt{5} \rightarrow \text{Ln}\, x = \text{Ln} \frac{6^3}{\sqrt{5}} \Rightarrow \boxed{x = \frac{6^3}{\sqrt{5}}}$

PROBLEMA 10.- Expresa como un solo logaritmo cada una de las siguientes expresiones:

a) 2·logA+3·logB b) 1+2·logA-2·log B c) ½ log x – 2 log y + log z

a) $2 \cdot \log A + 3 \log B = \log A^2 + \log B^3 = \log A^2 \cdot B^3$

b) $1 + 2 \cdot \log A - 2 \log B = \log 10 + \log A^2 - \log B^2 = \log \frac{10 A^2}{B^2}$

c) $\frac{1}{2} \log x - 2 \log y + \log z = \log \sqrt{x} - \log y^2 + \log z =$

$= \log \frac{\sqrt{x}}{y^2} \cdot z$

PROBLEMA 11.- Resuelve las siguientes ecuaciones logarítmicas:

1) log(2x) = -1

2) $\log x^2 = 1/2$

3) log (3x) = 2

4) log x = -1

5) log x = 3/2

6) $\log (3x^2) = -2$

7) log x + log 50 = log 100

8) $\log x^3 = \log 6 + 2 \log x$

9) log x = 1 + log (22-x)

10) 2 log x – log (x-16) = 2

11) 5 log (2x) = 20

12) $\log \left(\frac{2x-4}{5} \right) = 2$

13) log x + log 20 = 3

14) $\log (x+1)^2 = 2$

15) 3 log (5x) = -9

16) $3 \log x + 2 \log x^2 = \log 128$

17) $\log \frac{10}{x} = 2-2\log x$

18) $\log \frac{x}{2} = 1 + \log (21-x)$

19) $\log (7x+15) - \log 5 = 1$

20) $2\log x - \log (x^2-2x+6)=0$

21) $2\log x = \log (10-3x)$

22) $\log (2x-3) + \log (3x-2) = 2 - \log 25$

23) $(x^2-x+3)\log 4 = 3\log \frac{1}{4}$

24) $\log 8 + (x^2-5x+7)\log 3 = \log 24$

25) $\log (5x+4) - \log 2 = \frac{1}{2}\log (x+4)$

26) $\frac{\log(35-x^3)}{\log(5-x)} = 3$

27) $\frac{\log 2+\log(11-x^2)}{\log(5-x)}=2$

28) $\log (25-x^3) - 3\log (4-x)=0$

29) $\log x = \frac{2-\log x}{\log x}$

30) $log_5 x + \frac{log_5 125}{log_5 x} = \frac{7}{2}$

1) $\log 2x = -1 \rightarrow 2x = 10^{-1} \rightarrow 2x = 0,1 \Rightarrow x = \frac{0,1}{2} = \boxed{0,05}$

2) $\log x^2 = 1/2 \rightarrow x^2 = 10^{1/2} \rightarrow \boxed{x = \sqrt[4]{10}}$

3) $\log 3x = 2 \rightarrow 3x = 10^2 \rightarrow \boxed{x = 100/3}$

4) $\log x = -1 \rightarrow x = 10^{-1} \rightarrow \boxed{x = 0,1}$

5) $\log x = 3/2 \rightarrow x = 10^{3/2} \rightarrow \boxed{x = 10\sqrt{2}}$

6) $\log (3x^2) = -2 \rightarrow 3x^2 = 10^{-2} \rightarrow x^2 = \frac{100}{3} \rightarrow \boxed{x = \frac{10}{\sqrt{3}}}$

7) $\log x + \log 50 = \log 100 \rightarrow \log x = \log 100 - \log 50$

$\log x = \log \frac{100}{50} \rightarrow \log x = \log 2 \rightarrow \boxed{x = 2}$

8) $\log x^3 = \log 6 + 2\cdot\log x \rightarrow \log x^3 = \log 6 + \log x^2$

$\log x^3 = \log 6x^2 \rightarrow x^3 = 6x^2 \rightarrow x^3 - 6x^2 = 0$

$x^2(x-6) = 0$ $\begin{cases} x^2 = 0 \Rightarrow x = 0 \text{ NO VÁLIDA} \\ x-6 = 0 \Rightarrow \boxed{x = 6} \end{cases}$

8) $\log x^3 = \log 6 + 2\cdot\log x \rightarrow \log x^3 = \log 6 + \log x^2$

$\log x^3 = \log 6x^2 \rightarrow x^3 = 6x^2 \rightarrow x^3 - 6x^2 = 0$

$x^2(x-6) = 0$ $\begin{cases} x^2 = 0 \Rightarrow x = 0 \text{ NO VÁLIDA} \\ x - 6 = 0 \Rightarrow \boxed{x = 6} \end{cases}$

10) $2\cdot\log x - \log(x-16) = 2 \rightarrow \log x^2 - \log(x-16) = \log 100$

$\log \frac{x^2}{x-16} = \log 100 \Rightarrow \frac{x^2}{x-16} = 100 \rightarrow x^2 = 100x - 1600$

$x^2 - 100x + 1600 = 0 \rightarrow x = \frac{100 \pm \sqrt{100^2 - 4\cdot 1600}}{2} =$

$= \frac{100 \pm \sqrt{3600}}{2} = \frac{100 \pm 60}{2} = \frac{160}{2} = 80$ $\boxed{x = 80}$

$= \frac{40}{2} = 20$ $\boxed{x = 20}$

11) $5\cdot\log 2x = 20 \rightarrow \log 2x = \frac{20}{5} \rightarrow \log 2x = 4$

$\log 2x = \log 10^4 \rightarrow 2x = 10000 \rightarrow \boxed{x = 5000}$

12) $\log\left(\frac{2x-4}{5}\right) = 2 \rightarrow \log\left(\frac{2x-4}{5}\right) = \log 100 \rightarrow \frac{2x-4}{5} = 100$

$2x - 4 = 500 \rightarrow 2x = 500 + 4 \rightarrow x = \frac{504}{2} \rightarrow \boxed{x = 252}$

13) $\log x + \log 20 = 3 \rightarrow \log 20x = \log 1000$

$20x = 1000 \rightarrow x = \frac{1000}{20} \rightarrow \boxed{x = 50}$

14) $\log(x+1)^2 = 2 \rightarrow \log(x+1)^2 = \log 100 \rightarrow (x+1)^2 = 100$

$x^2 + 2x + 1 = 100 \rightarrow x^2 + 2x - 99 = 0$

$x = \frac{-2 \pm \sqrt{4 + 4\cdot 99}}{2} = \frac{-2 \pm \sqrt{400}}{2} = \frac{-2 \pm 20}{2}$ $\begin{cases} \frac{-2-20}{2} = -11 \\ \frac{-2+20}{2} = 9 \end{cases}$

$\boxed{x = -11}$ $\boxed{x = 9}$

15) $3\cdot\log 5x=-9 \rightarrow \log 5x=-\frac{9}{3}=-3 \rightarrow 5x=10^{-3} \rightarrow \boxed{x=\frac{1}{5\cdot 10^3}}$

16) $3\log x+2\log x^2=\log 128 \rightarrow \log x^3+\log x^4=\log 128$

$\log x^3\cdot x^4=\log 128 \rightarrow \log x^7=\log 128 \rightarrow x^7=128$

$x^7=2^7 \rightarrow \boxed{x=2}$

17) $\log\frac{10}{x}=2-2\log x \rightarrow \log\frac{10}{x}=\log 100-\log x^2$

$\log\frac{10}{x}=\log\frac{100}{x^2} \rightarrow \frac{10}{x}=\frac{100}{x^2} \rightarrow 10x^2=100x$

$10x^2-100x=0 \rightarrow 10x(x-10)=0$ $\begin{cases} x=0 \text{ NO VALIDA} \\ \boxed{x=10} \end{cases}$

18) $\log\frac{x}{2}=1+\log(21-x) \rightarrow \log\frac{x}{2}=\log 10+\log(21-x)$

$\log\frac{x}{2}=\log(210-10x) \rightarrow \frac{x}{2}=210-10x \rightarrow x=420-20x$

$20x+x=420 \rightarrow 21x=420 \rightarrow x=\frac{420}{21}=20 \rightarrow \boxed{x=20}$

19) $\log(7x+15)-\log 5=1 \rightarrow \log\frac{7x+15}{5}=\log 10 \rightarrow \frac{7x+15}{5}=10$

$7x+15=50 \rightarrow 7x=50-15 \rightarrow 7x=35 \rightarrow \boxed{x=5}$

20) $2\cdot\log x-\log(x^2-2x+6)=0 \rightarrow \log x^2-\log(x^2-2x+6)=\log 1$

$\frac{x^2}{x^2-2x+6}=1 \rightarrow x^2=x^2-2x+6 \rightarrow 2x=6 \rightarrow x=\frac{6}{2} \rightarrow \boxed{x=3}$

21) $2\cdot\log x=\log(10-3x) \rightarrow \log x^2=\log(10-3x)$

$x^2=10-3x \rightarrow x^2+3x-10=0 \rightarrow x=\frac{-3\pm\sqrt{9-4\cdot 1\cdot(-10)}}{2\cdot 1}$

$x=\frac{-3\pm\sqrt{9+40}}{2}=\frac{-3\pm\sqrt{49}}{2}=\frac{-3\pm 7}{2}$ $\begin{cases} \frac{-3+7}{2}=\frac{4}{2}=2 \rightarrow \boxed{x=2} \\ \frac{-3-7}{2}=\frac{-10}{2}=-5 \text{ NO VALIDA} \end{cases}$

22) $\log(2x-3)+\log(3x-2)=2-\log 25$

$\log(2x-3)\cdot(3x-2)=\log 100-\log 25$

$\log(6x^2-4x-9x+6)=\log\frac{100}{25}$

$6x^2-13x+6=4 \rightarrow 6x^2-13x+2=0 \rightarrow x=\frac{13\pm\sqrt{13^2-4\cdot6\cdot2}}{2\cdot6}$

$x=\frac{13\pm\sqrt{169-48}}{12}=\frac{13\pm\sqrt{121}}{12}=\frac{13\pm11}{12}$ $\begin{cases}\frac{13+11}{12}=2 \rightarrow \boxed{x=2}\\ \frac{13-11}{12}=\frac{2}{12}=\frac{1}{6}\end{cases}$

COMPROBAMOS: $\log\left(2\cdot\frac{1}{6}-3\right)+\log\left(3\cdot\frac{1}{6}-2\right)=$

$\log(-8/3)+\log(-3/2) \Rightarrow$ NO VÁLIDA

23) $(x^2-x+3)\cdot\log 4=3\cdot\log 1/4$

$\log 4^{x^2-x+3}=\log\left(\frac{1}{4}\right)^3 \rightarrow 4^{x^2-x+3}=(4^{-1})^3=4^{-3}$

$x^2-x+3=-3 \rightarrow x^2-x+6=0 \rightarrow x=\frac{1\pm\sqrt{1-12}}{2}$ SIN SOLUCIÓN

24) $\log 8+(x^2-5x+7)\cdot\log 3=\log 24$

$\log 8+\log 3^{x^2-5x+7}=\log 24 \rightarrow \log 8\cdot3^{x^2-5x+7}=\log 24$

$8\cdot3^{x^2-5x+7}=24 \rightarrow 3^{x^2-5x+7}=\frac{24}{8}=3$

$x^2-5x+7=1 \rightarrow x^2-5x+7-1=0 \rightarrow x^2-5x+6=0$

$$x=\frac{5\pm\sqrt{5^2-4\cdot1\cdot6}}{2\cdot1}=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm\sqrt{1}}{2}=\frac{5\pm1}{2}\begin{cases}\frac{5+1}{2}=\frac{6}{2}=3\\ \frac{5-1}{2}=\frac{4}{2}=2\end{cases}$$

$\boxed{x=2}$ $\boxed{x=3}$

25) $\log(5x+4)-\log 2=\frac{1}{2}\log(x+4)$

$$\log\frac{5x+4}{2}=\log(x+4)^{1/2}\rightarrow\left(\frac{5x+4}{2}\right)=\left(\sqrt{x+4}\right)^2$$

$$\frac{25x^2+4x+16}{4}=x+4\rightarrow 25x^2+4x+16=4x+16$$

$$25x^2=0\Rightarrow\boxed{x=0}$$

26) $\frac{\log(35-x^3)}{\log(5-x)}=3\rightarrow\log(35-x^3)=3\cdot\log(5-x)$

$$\log(35-x^3)=\log(5-x)^3\rightarrow 35-x^3=(5-x)^3$$

$$35-x^3=5^3-3\cdot5^2x+3\cdot5x^2-x^3$$

$$-15x^2+75x-125+35=0\rightarrow 15x^2-75x+90=0$$

$$x^2-5x+6=0\rightarrow x=\frac{5\pm\sqrt{25-4\cdot1\cdot6}}{2\cdot1}=\frac{5\pm1}{2}\begin{cases}\frac{5+1}{2}=3\\ \frac{5-1}{2}=2\end{cases}$$

$\boxed{x=2}$ $\boxed{x=3}$

27) $\frac{\log 2+\log(11-x^2)}{\log(5-x)}=2\rightarrow\log(22-2x^2)=2\cdot\log(5-x)$

$$\log(22-2x^2)=\log(5-x)^2\rightarrow 22-2x^2=(5-x)^2$$

$$22-2x^2=25-10x+x^2\rightarrow -3x^2+10x-3=0$$

$$x=\frac{-10\pm\sqrt{100-4(-3)(-3)}}{2(-3)}=\frac{-10\pm\sqrt{100-36}}{-6}=\frac{-10\pm\sqrt{64}}{-6}$$

$$x=\frac{-10\pm8}{-6}\begin{cases}\frac{-10+8}{-6}=\frac{-2}{-6}=1/3\\ \frac{-10-8}{-6}=\frac{-18}{-6}=3\end{cases}$$

$\boxed{x=1/3}$ $\boxed{x=3}$

28) $\log(25-x^3)-3\log(4-x)=0$

$\log(25-x^3)=\log(4-x)^3 \to 25-x^3=(4-x)^3$

$25-x^3=4^3-3\cdot4^2\cdot x+3\cdot4\cdot x^2-x^3 \to 12x^2-48x+64-25=0$

$12x^2-48x+39=0 \to 4x^2-16x+13=0$

$x=\frac{16\pm\sqrt{16^2-4\cdot4\cdot13}}{2\cdot4}=\frac{16\pm\sqrt{256-208}}{8}=\frac{16\pm\sqrt{48}}{8}$

$\boxed{x=\frac{4-\sqrt{3}}{2}}$ $\boxed{x=\frac{4+\sqrt{3}}{2}}$

29) $\log x=\frac{2-\log x}{\log x} \to \log^2 x=2-\log x \to \log^2 x+\log x-2=0$

$\log x=t \to t^2+t-2=0 \to t=\frac{-1\pm\sqrt{1+4\cdot2}}{2\cdot1}=\frac{-1\pm3}{2}$

$t_1=\frac{-1-3}{2}=\frac{-4}{2}=-2$ $\log x=-2 \to \boxed{x=10^{-2}=0{,}01}$

$t_2=\frac{-1+3}{2}=\frac{2}{2}=1$ $\log x=1 \to x=10^1 \Rightarrow \boxed{x=10}$

30) $\log_5 x+\frac{\log_5 125}{\log_5 x}=\frac{7}{2} \to 2\log_5^2 x+2\log_5 125=7\log_5 x$

$2\log_5^2 x-7\log_5 x+2\cdot3=0 \to \log_5 x=t$

$2t^2-7t+6=0 \to t=\frac{7\pm\sqrt{49-4\cdot2\cdot6}}{2\cdot2}=\frac{7\pm\sqrt{49-48}}{4}$

$t=\frac{7\pm\sqrt{1}}{4}$ $t_1=\frac{7+1}{4}=\frac{8}{4}=2 \to \log_5 x=2 \to x=5^2 \to \boxed{x=25}$

$t_2=\frac{7-1}{4}=\frac{6}{4}=\frac{3}{2} \to \log_5 x=3/2 \to x=5^{3/2} \to \boxed{x=5\sqrt{5}}$